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Nonlinear model predictive control of a pH neutralization process based on Wiener–Laguerre model

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ABSTRACT

In this paper, Laguerre filters and simple polynomials are used respectively as linear and nonlinear parts of a Wiener structure. The obtained model structure is the so-called Wiener–Laguerre model. This model is used to evaluate identification of a pH neutralization process. Then the model is used in a nonlinear model predictive control framework based on the sequential quadratic programming (SQP) algorithm. Various orders of Laguerre filters and nonlinear polynomials are tested, and the results are compared for the validation of these models. Validation results for various orders suggest that in order to have a good trade-off between simplicity of the model and its corresponding fitness, a second order nonlinear polynomial along with two Laguerre filters may be selected. The fitness of this model according to variance account for (VAF) criterion is 92.32%, which is completely acceptable for nonlinear model predictive control applications. Then the identified Wiener–Laguerre model is used for nonlinear model predictive control and the results are compared with model predictive control in which just Wiener model was used for identification. It is shown that the use of the Wiener–Laguerre structure improves the quality of modeling together with the rate of convergence of SQP in a reasonable time. Furthermore, these results are also compared with the performance of a linear model predictive controller based on Laguerre model to provide a fair comparison between linear and nonlinear systems.

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1. Introduction

Model predictive control (MPC) is one of the most successful controllers in process industries. MPC describes a class of computer control algorithms that control the future behavior of a plant through the use of an explicit process model [1]. Therefore, the core of the MPC algorithm is a dynamic model. Until recently, industrial applications of MPC have relied on linear dynamic models even though most processes are nonlinear. MPC based on linear models are acceptable when the process operates at a single set point and the primary use of the controller is the rejection of disturbances. Many chemical processes, however, do not operate at a single set point, and they are often required to operate at different set points depending on the grade of the product to be produced. Because these processes make transitions over the nonlinearity of the system, linear MPC often results in poor control performance. To properly control these processes, a nonlinear model is needed in the MPC algorithm. Process industries need predictive controllers that are low cost, easy to setup, and account for plant nonlinearity as well as modeling uncertainties. Therefore, it is necessary to obtain a suitable nonlinear modeling technique that can be easily used in a nonlinear MPC framework.

Selection of a suitable structure of a nonlinear model to represent system dynamics is a crucial step in the development of a nonlinear MPC (NMPC) scheme. A number of researchers and commercial companies have developed nonlinear models using a variety of technologies, including first-principle [2] and empirical approaches (i.e., nonlinear black-box models) [3–5]. The first-principle models are valid globally and can predict system dynamics over the entire operating range. However, development of a reliable first-principle model is a difficult and time-consuming task. On the other hand, the nonlinear black-box models have certain advantages over the first-principle models in terms of development time and efforts. Thus, from a practical viewpoint, development of an NMPC scheme based on a nonlinear black-box model is a more attractive choice.

In the development of a nonlinear black-box model, selection of a suitable model structure that can capture nonlinear dynamics over a wide operating range is not easy [1]. Different black-box model structures are nonlinear autoregressive with exogenous inputs (NARX) models [3], Volterra series expansion models and

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block oriented models (Hammerstein and Wiener structures) [6], and artificial neural networks (ANN) [5]. The determination of model order and model structure of a general NARX model (i.e. which terms are important and cannot be ignored) is a difficult task [1].

Volterra series models can be used to model a wide class of nonlinear systems. However, these models are non-parsimonious in parameters. Su and McAvoy [5] showed that recurrent neural networks (RNN) are better suited for the development of NMPC schemes. These models have output error structure and generate better long-range predictions than feed forward neural networks (FNN) that have been widely employed in process control applications. Development of RNN models is, however, considerably more difficult than development of FNN models. Therefore, it is necessary to evolve a scheme for the development of a black-box model in which the model structure can be selected relatively easily and the resulting model is valid over a wide operating range.

Block oriented models such as Wiener models are well-known in NMPC because of their simplicity and capability in modeling nonlinear systems, specially those that have linear dynamic and nonlinear output mapping. Wiener models have the capability of approximating, with arbitrary accuracy, any fading memory nonlinear time invariant system [7], and they have been successfully used to model several nonlinear systems encountered in the process industry, such as distillation columns [8] and pH processes [9].

The linear convolution type models have been widely used in MPC implementation as these models do not require model order and time delay to be specified, and modeling of MIMO systems is considerably easy using this representation [10]. A linear black-box modeling technique that has received increasing attention in the past decade is the Laguerre Series approximation [11,12]. A Laguerre series model can be looked upon as a compact representation of these convolution type models.

Advantages of the linear Laguerre model can be summarized as follows:

1. Laguerre models do not need any explicit knowledge about system time constant and time delay for model development.
2. Unlike the convolution type models that require large number of coefficients, a good approximation can be obtained with a small number of model coefficients for asymptotically stable systems due to orthogonality property of Laguerre polynomials.
3. The estimates of the Laguerre coefficients are unbiased even for a truncated series.

The use of such orthonormal filters in combination with a memoryless nonlinear map referred to hereafter as a Wiener–Laguerre model was originally proposed by Wiener [13].

The Wiener–Laguerre model can be looked upon as a representation of the Volterra series model that is parsimonious in parameters. Dumont et al. [14] has used a model of this type for developing an adaptive predictive control scheme for controlling SISO nonlinear systems. Sentoni et al. [15] used ANNs for constructing a nonlinear state output map. Saha et al. [13] used quadratic polynomials as well as ANN for constructing a nonlinear output map and used these models in nonlinear MPC formulations. The output of the Laguerre-polynomial model compares quite well with the output of pH neutralization process in the training data set; however, this model completely fails to predict the plant behavior when the validation data set is used [13].

In this paper, a nonlinear pH process is identified and controlled in its full range operating conditions that may happen in real applications. A Wiener–Laguerre structure is selected for identification with polynomial nonlinearity. The identification test is designed based on a GMN [16] signal, which is recommended for identification of nonlinear processes in industries. In order to keep the model as simple as possible, and also efficient for nonlinear model predictive control designs, different degrees of Laguerre filter and polynomial orders are used and compared. These are chosen, based on simulation results, in a trade-off between simplicity of the model and its corresponding fitness. Based on this model, a nonlinear model predictive controller is designed for a proper operation of the pH process in different set points, and the results are compared with a linear model predictive controller based on a linear Laguerre model. The performance of the controller based on the identified Wiener–Laguerre model shows that this model presents better prediction capabilities in comparison with the identified linear Laguerre model. Moreover, the MPC based on the Wiener–Laguerre model outperforms the MPC based on the Wiener model, particularly when the system is operating away from the nominal operating conditions.

This paper is organized in four sections. After this introduction, Laguerre filter networks and identification of nonlinear Wiener–Laguerre models are discussed in Section 2. This section is followed by the formulation of NMPC in Section 3, and a simulation study on pH neutralization is discussed in Section 4. Conclusions are summarized in Section 5.

2. Identification of Wiener–Laguerre models

Let us consider a SISO linear system, modeled by a Laguerre filter network and represented as follows:

$$\hat{y}(z) = \left(\sum_{i=1}^{N} c_i L_i(z) \right) u(z) \tag{1}$$

where

$$L_i(z) = \sqrt{(1-a^2)T_s} \frac{(1-az)^{i-1}}{(z-a)^i} \tag{2}$$

Here $L_i(z)$ denotes the ith order Laguerre filter, N the number of Laguerre filters used for model development, a $(-1 < a < 1)$ the Laguerre filter parameter, T_s the sampling interval, $\hat{y}(z)$ the model output, and $u(z)$ is the manipulated input.

Defining the state vector as

$$L(k) = [l_1(k), l_2(k), \ldots, l_N(k)]^T \tag{3}$$

where $l_i(k)$ represents the output from ith order Laguerre filter at kth sampling instant to the input $u(k)$, a discrete state-space realization of the Laguerre filter network can be obtained as follows[12]:

$$L(k+1) = \Phi(a)L(k) + \Gamma(a)u(k) \tag{4}$$

where $u(k)$ is the system input, $\Phi(a)$ is an $N \times N$ lower triangular matrix defined by

$$\Phi(a) = \begin{bmatrix} a & 0 & 0 & 0 & 0 \\ (1-a^2) & a & 0 & 0 & 0 \\ -a(1-a^2) & (1-a^2) & a & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & 0 \\ (-1)^N a^{N-2}(1-a^2) & (-1)^{N-1} a^{N-3}(1-a^2) & \cdots & \cdots & a \end{bmatrix} \tag{5}$$

and $\Gamma(a)$ is an N dimensional vector as follows:

$$\Gamma(a) = \left[\sqrt{(1-a^2)T_s}, -a\sqrt{(1-a^2)T_s}, \ldots, (-a)^{N-1}\sqrt{(1-a^2)T_s} \right]^T \tag{6}$$

For the linear model given by (1), the output can be expressed as the weighted sum of the states by

$$\hat{y}(k) = C^T L(k) \tag{7}$$

where elements of C are Laguerre filter coefficients, that is

$$C = [c_1, c_2, \ldots, c_N]^{\mathrm{T}}. \tag{8}$$

For developing a Wiener–Laguerre SISO nonlinear model, a nonlinear state-output map can be constructed so that the model output is represented as

$$\hat{y}(k) = \Psi[x(k)] \tag{9}$$

where $\psi(\cdot)$: $R^N \rightarrow R$ is a memoryless nonlinear function.

Here for simplicity of the model the nonlinear map $\psi(\cdot)$ is selected as a polynomial function of the elements of state vector. The resulting SISO symmetric Wiener–Laguerre model can be represented by

$$L(k+1) = \Phi(a)\,L(k) + \Gamma(a)\,u(k) \tag{10}$$

$$\hat{y}(k) = h_0 + \sum_{i=1}^{N} h_i l_i(k) + \sum_{i=1}^{N}\sum_{j=i}^{N} h_{ij} l_i(k) l_j(k) + \cdots + \sum_{i=1}^{N}\sum_{j=i}^{N} \cdots \sum_{k=r}^{N}\sum_{m=k}^{N} h_{ij\ldots km} l_i(k) l_j(k) \ldots l_k(k) l_m(k) \tag{11}$$

The key step in developing the Laguerre part of this model is to estimate the Laguerre filter parameter a, and select the number of Laguerre networks N. The step response data can be used to generate a meaningful initial guess for the filter parameter a. If T is the system time constant, the discrete pole of the Laguerre filter can be obtained as

$$a = e^{-T_s/T} \tag{12}$$

We have a strong preference for a pre-chosen real pole (i.e. Laguerre filter parameter) rather than obtaining it by optimization since it improves the speed and accuracy of the estimation algorithm. In exchange, this choice can lead to a slightly larger number of Laguerre filters required to model, for instance, under-damped second order dynamics.

The number of Laguerre filters and Volterra kernels N was chosen so that the output of the system and that of the model were best fitted according to the variance account for (VAF) criterion:

$$\mathrm{VAF} = \max\left\{1 - \frac{\mathrm{var}\{y - \hat{y}\}}{\mathrm{var}\{y\}}, 0\right\} \times 100\% \tag{13}$$

In (13) $y = \{y_k\}_{k=1}^{N_s}$ denotes the real output sequence, $\hat{y} = \{\hat{y}_k\}_{k=1}^{N_s}$ denotes the model output sequence, and var$\{\cdot\}$ denotes the variance of a quasi-stationary signal.

From (11) the output can be written as

$$\hat{y}(k) = H^{\mathrm{T}}\varphi(k) + \varepsilon(k) \tag{14}$$

where $\varepsilon(k)$ is the equation error and

$$H = [h_0, h_1, \ldots, h_R]^{\mathrm{T}}. \tag{15}$$

In (15), index R is calculated for a symmetric P-order Volterra filter by

$$R = \sum_{i=1}^{P} \binom{N+i-1}{i-1}. \tag{16}$$

Besides, the regression vector $\varphi(k)$ is defined by

$$\varphi(k) = [1\, l_1, \ldots, l_N, l_1^2\, l_1\, l_2, \ldots, l_N^2, l_1^3\, l_1\, l_2\, l_3, \ldots, l_N^3, \ldots, l_N^M]^{\mathrm{T}}. \tag{17}$$

After determining the structure of the proposed model, identification is performed using the least squares criterion.

3. Nonlinear model predictive formulation

In a typical MPC formulation, an explicit dynamic model is used at each sampling instant for predicting the future behavior of the plant over a finite number of future time steps, say P, which is called the prediction horizon. A set of M (called the control horizon) future manipulated input moves, $u(k/k)$, $u(k+1/k)$, …, $u(k+M-1/k)$, are determined by optimization with the objective of optimizing the future behavior of plant while taking into consideration the operating constraints.

Thus, given a sequence of future control moves, i.e., $u(k/k)$, $u(k+1/k)$, …, $u(k+M-1/k)$, the P step ahead open loop output prediction can be written as follows:

$$L(k+j+1\,|\,k) = \Phi(a)L(k+j\,|\,k) + \Gamma(a)u(k+j\,|\,k), \quad j = 1, \ldots, P-1 \tag{18}$$

$$\hat{y}(k+j\,|\,k) = \Psi[L(k+j\,|\,k)] \tag{19}$$

where

$$u(k+M\,|\,k) = \ldots = u(k+P-1\,|\,k) = u(k+M-1) \tag{20}$$

In order to take into account the plant model mismatch and unmeasured slightly varying disturbances, we assume that the discrepancy between the model output and the process output is due to additive step disturbances in the output that persist over the prediction horizon. Thus, similar to the linear dynamic matrix control scheme, a mismatch correction term is incorporated in the prediction model as follows:

$$y_c(k+j\,|\,k) = \hat{y}(k+j\,|\,k) + d(k\,|\,k), \quad j = 1, 2, \ldots, P \tag{21}$$

$$d(k\,|\,k) = y(k) - \hat{y}(k\,|\,k-1) \tag{22}$$

where $y(k)$ represents the measured plant output at the kth instant, and $\hat{y}(k/k-1)$ represents the model output at the kth instant using input sequence up to time $k-1$. Although simplistic, this type of unmeasured disturbance model approximates slowly varying disturbances and provides robustness to modeling error [17].

Now, given the future set point trajectory $\{y_{sp}(k+j/k); j = 1, 2, \ldots, P\}$ the controller design problem can be formulated as follows:

$$\min_{u(k/k)\ldots u(k+M-1/k)} \left\{ \sum_{j=1}^{P} \left\| E(k+j\,|\,k) \right\|_{Q(j)}^2 + \sum_{j=1}^{M} \left\| u(k+j\,|\,k) \right\|_{R(j)}^2 + \sum_{j=1}^{M} \left\| \Delta u(k+j\,|\,k) \right\|_{S(j)}^2 \right\} \tag{23}$$

subject to

$$u^L \le u(k+j\,|\,k) \le u^u, \quad j = 1, \ldots, M-1. \tag{24}$$

where

$$E(k+j\,|\,k) = y_{sp}(k+j\,|\,k) - y_c(k+j\,|\,k) \tag{25}$$

$$\Delta u(k+j\,|\,k) = u(k+j\,|\,k) - u(k+j-1\,|\,k) \tag{26}$$

In (23), $Q(j)$, $R(j)$ and $S(j)$ are positive semi-definite diagonal weighting matrices, and $||x||_z = \sqrt{x^{\mathrm{T}}Zx}$ denotes the weighted 2-norm of vector x. The weighting matrices $Q(j)$, $R(j)$ and $S(j)$, as well as the prediction horizon P and the control horizon M, are design parameters that must be tuned to provide the controller with a satisfactory performance.

The resulting nonlinear programming problem can be solved using any standard optimization technique such as successive

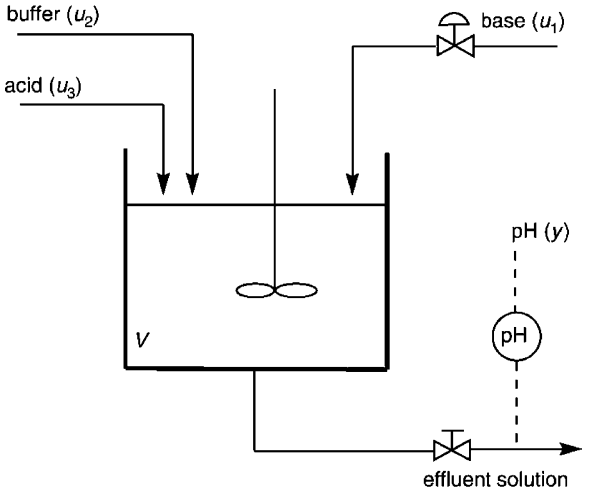

Fig. 1. Schematic representation of the pH neutralization process [18].

quadratic programming (SQP). The controller is implemented in a moving horizon framework, i.e., only $u(k/k)$ is implemented at each sampling instant, and the optimization is repeated at each sampling instant based on the updated information from the plant. In this work, the constrained optimization problem (23), resulting the optimal input $u(k/k)$ at every sampling instant, is solved using "fmincon" function in MATLAB optimization toolbox.

4. Simulation results

In this section, the proposed nonlinear modeling and predictive control are evaluated in simulation studies for the physical nonlinear model of a UCSB pH neutralization process [18].

4.1. pH neutralization process [18]

The considered pH neutralization process consists of an acid (HNO_3) stream, a base (NaOH) stream, and a buffer ($NaHCO_3$) stream that are mixed in a constant-volume (V) stirring tank. The process is schematically depicted in Fig. 1 [18].

Table 1
Nominal operating conditions

$u_3 = 16.60$ ml/s	$u_2 = 0.55$ ml/s
$u_1 = 15.55$ ml/s	$V = 2900$ ml
$W_{a1} = -3.05 \times 10^{-3}$ mol	$W_{a2} = -3 \times 10^{-2}$ mol
$W_{a3} = 3 \times 10^{-3}$ mol	$W_a = -4.32 \times 10^{-4}$ mol
$W_{b1} = 5 \times 10^{-5}$ mol	$W_{b2} = 3 \times 10^{-2}$ mol
$W_{b3} = 0$ mol	$W_b = 5.28 \times 10^{-4}$ mol
$pk_1 = 6.35$	$pk_2 = 10.25$
$y = 7.0$	

The inputs to the system are the base (volumetric) flow rate (u_1), the buffer flow rate (u_2), and the acid flow rate (u_3), while the output (y) is the pH of the effluent solution. The acid flow rate (u_3), as well as the volume (V) of the tank are assumed to be constant. Usually, the objective is to control the pH of the effluent solution by manipulating the base flow rate, despite the variations of the unmeasured buffer flow rate, which can be considered as unmeasured disturbance.

The model is highly nonlinear due to the implicit output equation, known as the titration curve given in (33). The dynamic model for the reaction invariants of the effluent solution (W_a, W_b) in state-space form is given by

$$\dot{x} = f(x) + g(x)u_1 + p(x)u_2 \tag{27}$$

$$h(x, y) = 0 \tag{28}$$

$$x \stackrel{\Delta}{=} [x_1, x_2]^T = [W_a, W_b]^T \tag{29}$$

$$f(x) = \left[\frac{u_3}{V}(W_{a3} - x_1), \frac{u_3}{V}(W_{b3} - x_2)\right]^T \tag{30}$$

$$g(x) = \left[\frac{1}{V}(W_{a1} - x_1), \frac{1}{V}(W_{b1} - x_2)\right]^T \tag{31}$$

$$p(x) = \left[\frac{1}{V}(W_{a2} - x_1), \frac{1}{V}(W_{b2} - x_2)\right]^T \tag{32}$$

$$h(x, y) = x_1 + 10^{y-14} - 10^{-y} + x_2 \frac{1 + 2 \times 10^{y-pk_2}}{1 + 10^{pk_1 - y} + 10^{y-pk_2}} \tag{33}$$

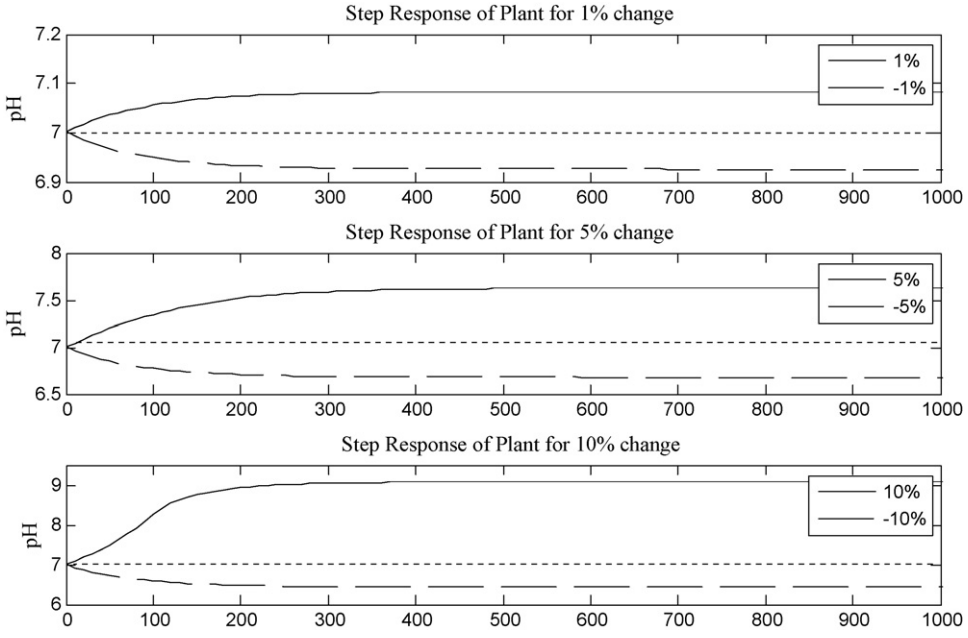

Fig. 2. Step responses of pH process.

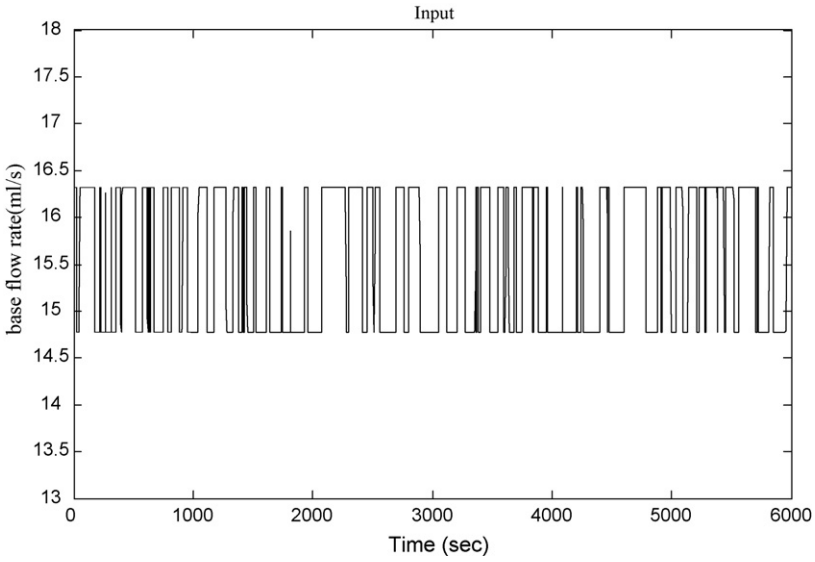

Fig. 3. GBN test signal.

Here, parameters pk_1 and pk_2 are the first and second disassociation constants of the weak acid H_2CO_3. The nominal operating conditions of the system are given in Table 1 for the sake of completeness.

4.2. Pre-test and identification test design

In order to develop a Laguerre model we should guess the filter parameter '*a*' at first. Therefore, we need to obtain a rough estimate of dominant process time constants through step tests. In a step test, the process is operating in open-loop without the model-based controller, each input is stepped separately, and step responses are recorded. The maximum step size can be determined according to process operation experience, and the step length should be longer than the settling time of the process. Step tests of pH neutralization process for one step up and one-step down with different step sizes are shown in Fig. 2. As it is clear from this figure, pH neutralization process as a nonlinear system shows different time constants between 100 and 160 s for step sizes up to ±10%.

Generalized binary noise (GBN) [16] around the nominal value of the base flow rate is used as the exciting signal for identification of the linear Laguerre model. Here, linear identification is carried out around ±5% of the base flow rate (u_1 = 15.55 ml/s) and switching time is calculated as

$$T_{sw} = \frac{T_s}{3} \tag{34}$$

where

$$T_s = 0.98T_{settling} \tag{35}$$

and $T_{settling}$ is the settling time of the system which is 160 s here according to the step response of ±5%. The GBN signal designed for identification of linear model is shown in Fig. 3.

Traditionally, pseudo-random binary sequences (PRBS) are used as the inputs to a system in order to produce representative sets of data to be analyzed. In theory, a PRBS excites the range of dynamics present in a system so that a dynamic model can be produced which contains these dynamics. This is not sufficient, however, for fitting

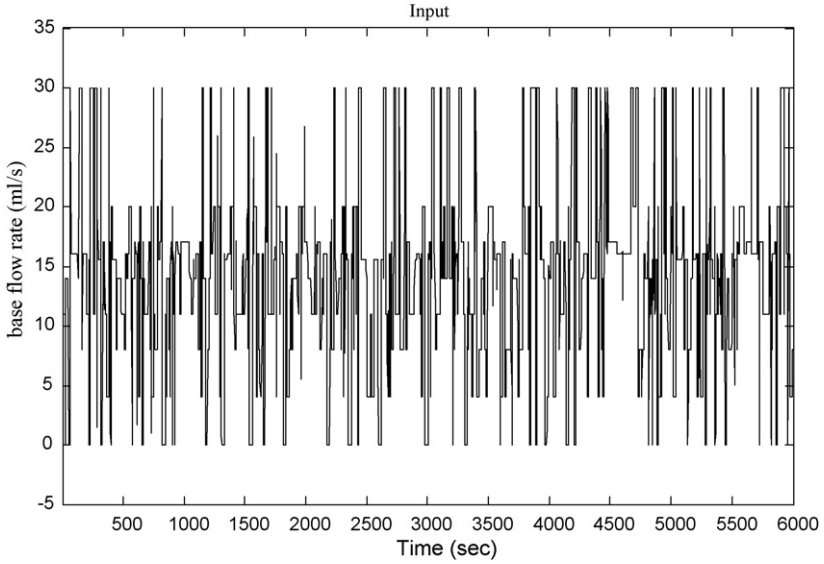

Fig. 4. GMN test signal.

a Wiener model. Since these models have nonlinear gains, an input signal must be used which also demonstrates the response of the system to a range of amplitude changes. A signal which satisfies this criterion is a GMN or a modified PRBS signal which, in addition to random frequency, also exhibits random amplitude changes. Generalized multi level noise (GMN) is a multilevel generalization of GBN [16]. Here a GMN is used as a test signal for the identification of a Wiener–Laguerre model. The switching time is designed as for the GBN test signal. Amplitude distribution is chosen so that it has 10 levels around the nominal value of the base flow rate to cover the whole operating region (between 0 and 30). These levels play an important role in the identification of Wiener–Laguerre model. Fig. 4 shows the GMN test signal.

4.3. Linear identification

The nonlinear analytical model (27) and (28) of the process is used to generate input–output data for the identification of a linear Laguerre model and also a Wiener–Laguerre model of the process.

At first, a linear Laguerre model is considered, and a generalized binary noise around the nominal value of the base flow rate (Fig. 3) is used as the exciting signal for identification of the linear Laguerre model. Buffer flow rate is kept constant at its nominal value (0.55 ml/s), and the acid flow rate is kept constant at 16.60 ml/s. The output of the system was corrupted with an additive Gaussian white noise with zero mean and standard deviation $\sigma = 0.001$ (S/N ratio = 10), in order to simulate a more realistic situation when measurement noise is present. The input–output data are plotted in Fig. 5. Output data has 6000 samples and is gathered with 10-s sampling time. The first 4500 samples are used for identification of the model and the rest of 1500 samples for validation purpose. Different orders of linear Laguerre filters are tested and the best one is selected according to the VAF criterion. Results are summarized in Table 2 [19].

It shows that when N increases, the fitness of the identified model increases too. However, only increasing N from $N = 1$ up to 2 improved the performance of the MPC controller, and this performance almost saturated for $N > 2$. Therefore $N = 2$ was chosen in a trade-off between compactness of the identified model and its corresponding fitness.

The result of validation for the estimated Laguerre model using validation data is shown in Fig. 6. The fitting is 94.32% according to the VAF criteria. Fig. 6 shows that a linear Laguerre model can capture the dynamic of the process but it cannot model its nonlinear gain. So adding a nonlinear mapping as the nonlinear gain seems to be necessary to improve the model accuracy.

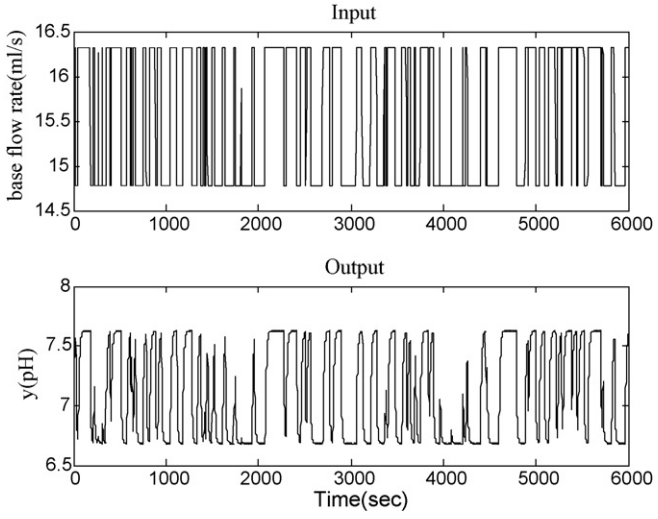

Fig. 5. Input–output data for linear identification.

Table 2
Comparison of identified linear models for different orders of Laguerre filters (noisy data)

	$N=1$	$N=2$	$N=3$	$N=4$	$N=5$
VAF	87.85	94.32	95.12	95.68	96.16
	$N=6$	$N=7$	$N=8$	$N=9$	$N=10$
VAF	96.78	97.07	97.24	97.26	97.35

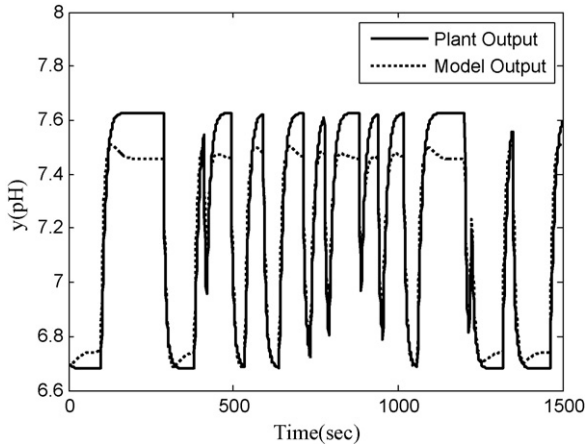

Fig. 6. True and estimated output for linear Laguerre model (validation data).

4.4. Nonlinear identification

In this case the excitement signal was a generalized multilevel noise (GMN) designed in Section 4.2. Again, buffer and acid flow rate were fixed at their nominal values. In order to simulate a more realistic situation of having measurement noise, a Gaussian white noise with zero mean and standard deviation $\sigma = 0.001$ (S/N ratio = 10) was added to the output of the system. The input–output data used for nonlinear identification are shown in Fig. 7. Output data has 6000 samples and are gathered with 10-s sampling time. Different orders of nonlinear mapping polynomial (P) and Laguerre filter (N) were tested and the results are summarized in Table 3. According to Table 3, for a simple choice of $P = 2$, the highest VAF value is obtained for $N = 2$. In this case, while keeping the model simple, we get enough accuracy for designing a nonlinear controller based on this model.

The result of validation for the estimated Wiener–Laguerre model is depicted in Fig. 8. The fitting according to the VAF criteria is 92.32%. When compared to the Laguerre model in Fig. 6, the

Table 3
Comparison of identified Wiener–Laguerre model for various orders of polynomial (P) and Laguerre filters (N)

	$N=1$	$N=2$	$N=3$	$N=4$	$N=5$
$P=2$	92.11	92.32	92.19	90.85	90.31
$P=3$	95.87	97.08	97.15	97.09	97.11
$P=4$	95.87	97.06	97.15	96.84	*96.44*
	$N=6$	$N=7$	$N=8$	$N=9$	$N=10$
$P=2$	89.65	89.15	88.71	85.15	81.78
$P=3$	96.96	96.94	96.92	96.67	95.67
$P=4$	95.86	92.89	87.73	50.36	*−529.06*

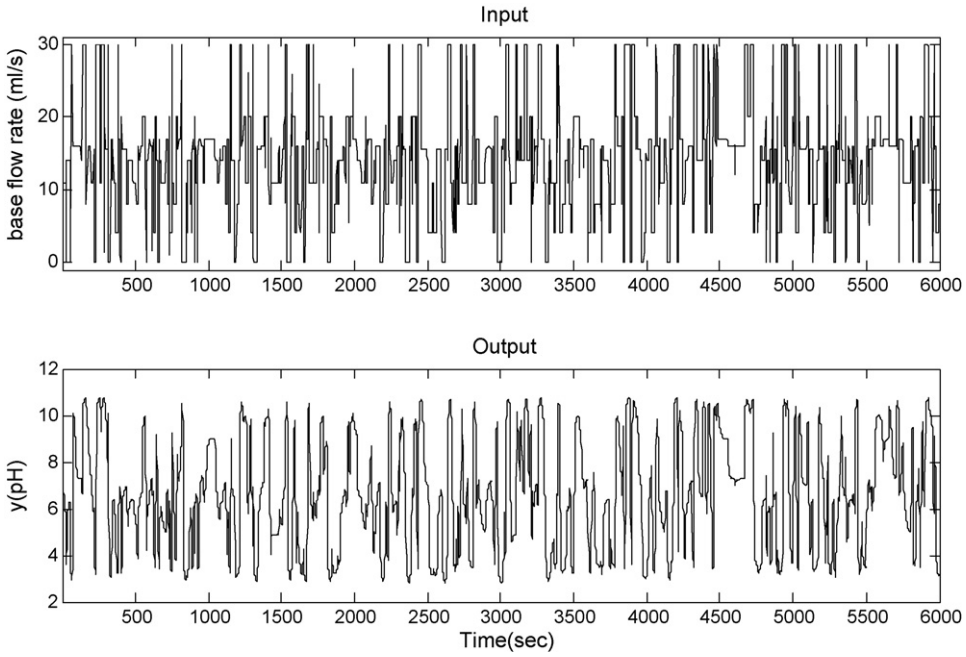

Fig. 7. Input–output data for nonlinear identification.

Wiener–Laguerre model can be seen to better model the nonlinear gain compared to the linear Laguerre model.

4.5. Model predictive control design

The linear model predictive control scheme is simulated using the "mpctool" in the MPC toolbox of MATLAB, and the linear Laguerre model identified in Section 4.3 is used for prediction.

Saturation constraints in the manipulated variables are imposed to take into account the minimum/maximum aperture of the valve regulating the base flow rate. A lower limit of 0 ml/s and an upper limit of 30 ml/s are chosen for this variable. The tuning parameters that have significant effects on both linear and nonlinear MPC performance are the prediction horizon, control horizon, sampling interval and penalty weighting matrices. The parameters used in the design of the MPC controller are tuned as follows:

1. The prediction horizon was set to 8 as a result of using different levels and comparing control performances.
2. A control horizon of two samples was found to provide a good control performance.
3. The weighting Q associated with the error from set point was selected two times greater than the weighting S associated with the input signal changes. Tuning parameters for linear MPC are shown in Table 4.

In Fig. 9, the simulation result with the MPC algorithm based on the linear Laguerre model identified in Section 4.3 is compared with the MPC based on a linear state-space model (identified using the N4SID algorithm) proposed in Ref. [20]. Also, the corresponding control signals are plotted in Fig. 10. It can be observed that the MPC based on the Laguerre model performs better than that based on the state-space model, when the operating region is far from the nominal operating conditions (pH 7).

4.6. Nonlinear model predictive control design

The NMPC scheme introduced in Section 3 is simulated using the Wiener–Laguerre model identified in Section 4.4. The optimization problem is solved using "fmincon" function in the optimization toolbox of MATLAB. Tuning parameters of the controller are shown in Table 5. The results are shown in Figs. 11 and 12, where they are compared with the results obtained from an MPC controller based on a Wiener model proposed in Ref. [20] and MPC controller of Section 4.5.

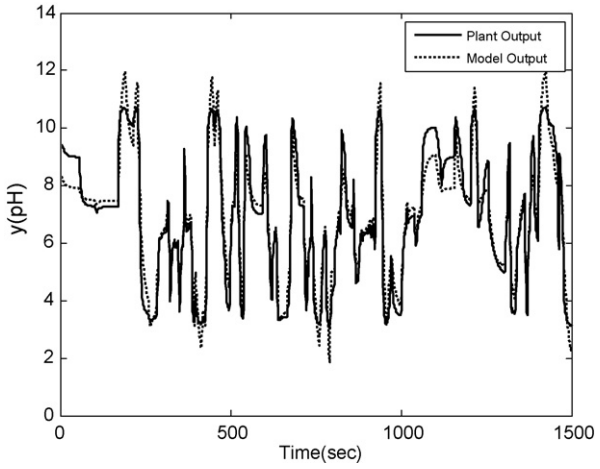

Fig. 8. True and estimated output of Wiener–Laguerre model (validation data).

Table 4
Tuning parameters of MPC controller

$Q = 100$
$R = 0$
$S = 50$
$P = 8$
$M = 2$
$u^L = 0$
$u^u = 30$

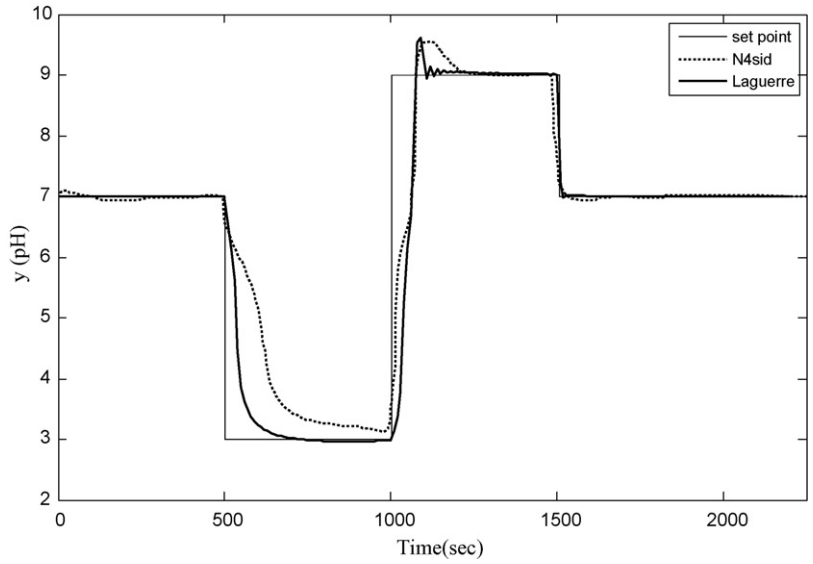

Fig. 9. The performance of the controller using Laguerre and state-space models.

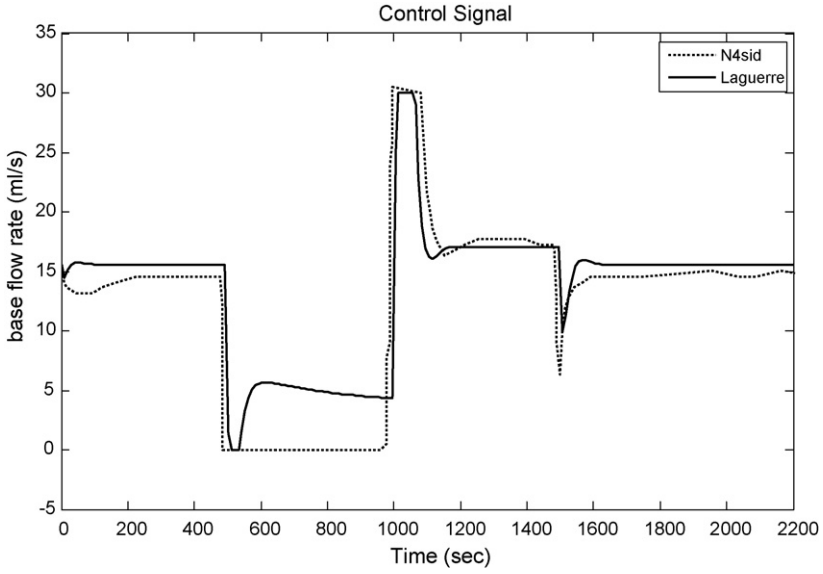

Fig. 10. Manipulated variable u_1 for Fig. 9.

Simulation results show that, for the considered application, the Wiener–Laguerre MPC performs slightly better than the MPC based on the linear Laguerre model, but it performs much better than that corresponding to the Wiener model presented in Ref. [20].

Fig. 12 shows the control efforts for both controllers, which are acceptable in a range from 0 up to 30, and Fig. 13 shows the CPU time. The maximum CPU time is in first seconds and never exceeds the system sample time (10 s). This fact ensures that in each sample time, the NMPC controller has enough time for its calculations and the optimization is performed in a reasonable time.

For the sake of comparison, Table 6 shows sum square error (SSE) in set point tracking for nonlinear MPC based on Wiener–Laguerre model and MPC based on Wiener model as well as Laguerre model. As can be seen, the NMPC based on Wiener–Laguerre shows better performance compared to the MPC based on Wiener model but is slightly better than MPC based on Laguerre model.

Table 5
Tuning parameters of NMPC controller

$Q = 100$
$R = 0$
$S = 5$
$P = 5$
$M = 2$
$u^L = 0$
$u^u = 30$

Table 6
SSE criteria for applied controllers in set-point tracking

Controller	SSE
NMPC (Wiener–Laguerre)	157.1854
MPC (Wiener)	429.9803
MPC (Laguerre)	199.671

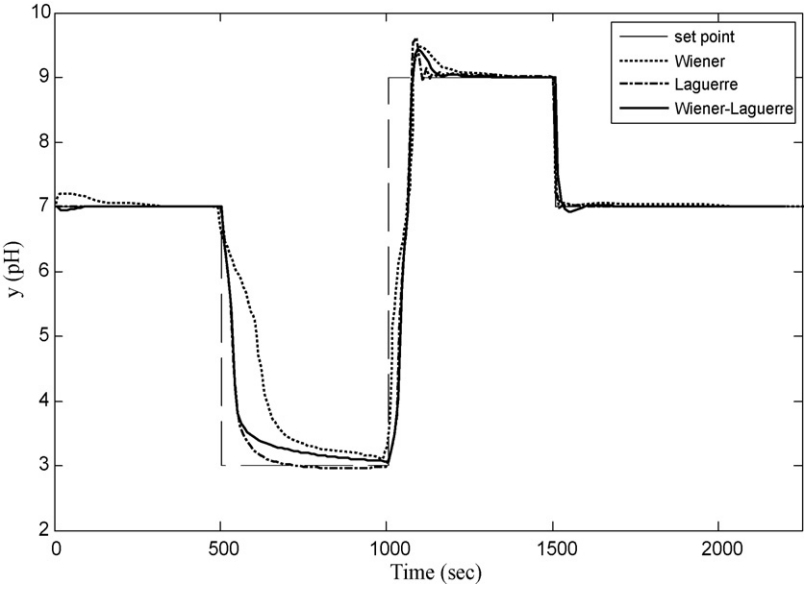

Fig. 11. NMPC based on Wiener–Laguerre in comparison with MPC based on Laguerre and Wiener models.

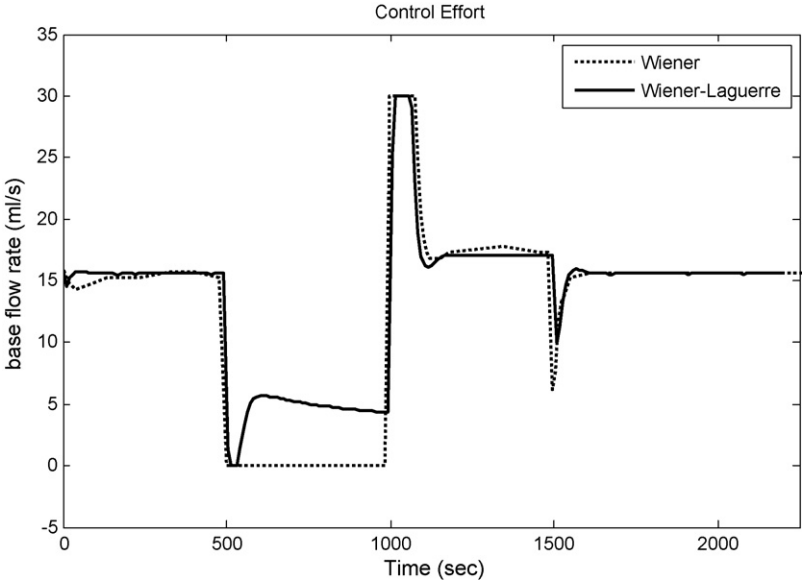

Fig. 12. Manipulated variable u_1 for Fig. 11.

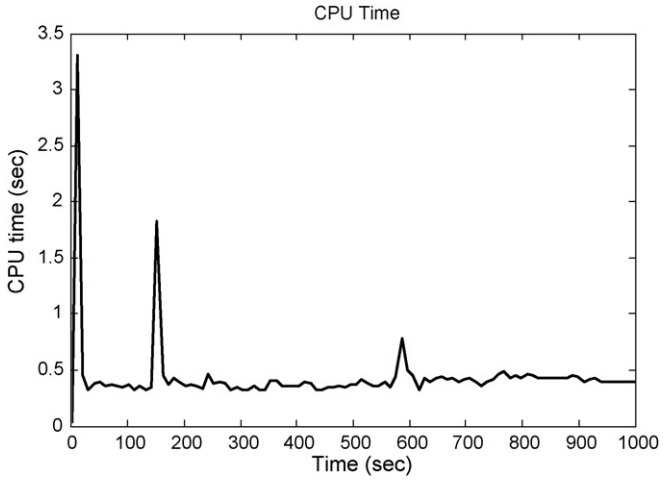

Fig. 13. CPU time.

5. Conclusions

Wiener models are frequently used for identification of nonlinear processes in nonlinear model predictive control systems. Laguerre filters are frequently used as the linear part of Wiener models resulting in the so-called Wiener–Laguerre model. This model structure was used for the identification of a highly nonlinear chemical process with the aim of being used in an NMPC controller. Various orders of the Laguerre network, as well as various polynomial orders were tested, and results are summarized in Table 3. Based on these results, the order of Laguerre network and also the polynomial order were chosen so that a good trade-off between the number of parameters and an acceptable VAF was obtained.

To show how Laguerre filters improve the modeling capability and performance of MPC controllers, the performance of an NMPC controller based on this Wiener–Laguerre model is compared with the performance of an MPC controller based on a Wiener model [20]. Simulation results show that the performance of the pro-

posed NMPC controller is slightly better than the linear one but it obviously outperforms the Wiener based MPC.

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